Resonance Frequency and Bandwidth of Rectangular Microstrip Antenna on Thick Substrate

A. K. Verma, *Member, IEEE,* and Nasimuddin

***Abstract*—We report an improved cavity model called modified Wolff model (MWM) to compute the resonance frequency of a rectangular patch antenna on the thick PTFE substrate ($0.037\lambda_g$–$0.229\lambda_g$). The MWM accounts for the effect of anisotropy and total losses on the resonance frequency, therefore it has a maximum deviation 2% against the experimental results. The previous cavity model, the multiport cavity model, and MOM based commercial software, Ensemble, compute resonance frequency with deviation between 4%–36%. Results on the bandwidth computed by these models have also been compared against the experimental results.**

***Index Terms*—Microstrip antennas, rectangular antennas, thick patch.**

I. INTRODUCTION

THE RECTANGULAR microstrip antenna shown in Fig. 1 is normally designed on the electrically thin substrates of the order of $h < 0.02\lambda_g$, where the λ_g is the guided wavelength in the substrate of relative permittivity given by $\lambda_g = \lambda_0/\sqrt{\varepsilon_r}$ at the operating wavelength λ_o. On the thick substrates between $0.037\lambda_g$–$0.229\lambda_g$, Chang *et al.* [1] and Kara [2] have obtained the resonance frequency of rectangular patches experimentally to achieve the bandwidth up to 22%. For such patches, the transmission line model and the cavity model compute the resonance frequency with average deviation between 3%–16% [1], [3], [4]. Usually, these experimental results have not been compared against any full-wave method. However, we have noted that the method of moments (MOM) based commercial software, Ensemble [5], provides deviation up to 12% for the patches of Chang *et al.* and up to 36% for the patches of Kara.

An improved form of the cavity model, called modified Wolff model (MWM), has been developed by Verma and co-workers [6], [7] to compute the resonance frequency of rectangular patches on the isotropic/anisotropic substrate. In this paper, apart from the effect of anisotropy in the substrate, we further account for the effect of all kind of losses in the patch on the resonance frequency. Thus, the present form of MWM computes accurately the resonance frequency of rectangular patches on electrically very thick substrate. The maximum deviation in the present model is 2% against the experimental results of Chang *et al.* and Kara.

Manuscript received July 16, 2001; revised November 13, 2001. This work was supported by the DOE (Present MIT), Government of India. The review of this letter was arranged by Associate Editor Dr. Ruediger Vahldieck.

The authors are with the Department of Electronic Science, University of Delhi South Campus, New Delhi, India.

Publisher Item Identifier S 1531-1309(02)01822-6.

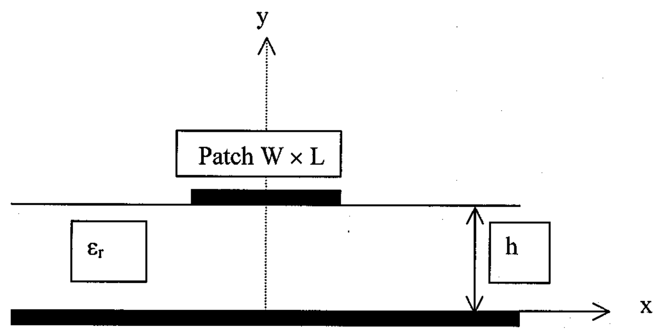

Fig. 1. Rectangular patch on thick substrate.

II. IMPROVED CAVITY MODEL (MODIFIED WOLFF MODEL)

We replace the uniaxial anisotropic substrate by the equivalent isotropic dielectric layer of equivalent relative permittivity $\varepsilon_{r\,eq}$, and equivalent substrate thickness h_{eq} [7]

$$\varepsilon_{r\,eq} = \sqrt{\varepsilon_{r\,xx}\varepsilon_{r\,yy}}, \qquad h_{eq} = h\sqrt{\frac{\varepsilon_{r\,xx}}{\varepsilon_{r\,yy}}} \tag{1}$$

where h is the actual thickness of substrate.

The above anisotropy relation has been obtained in the literature for the microstrip line on the anisotropic substrate by solving the boundary value problem [8]. The optic axis of the substrate is aligned to the geometrical axis of the patch.

The modified Wolff model (MWM) computes the resonance frequency of a rectangular patch on the lossy equivalent isotropic substrate [6], [7] by

$$f_r = \mathrm{Re}\left[\frac{v_0}{2\sqrt{\varepsilon^*_{r\,dyn}}}\left[\left(\frac{n}{L_{eff}}\right)^2 + \left(\frac{m}{W_{eff}}\right)^2\right]^{\frac{1}{2}}\right]. \tag{2}$$

In the normal cavity model, $\varepsilon^*_{r\,eff}$ or $\varepsilon^*_{r\,dyn}$ is taken as a real quantity. However, in the present case, the complex dynamic effective relative permittivity ($\varepsilon^*_{r\,dyn}$) takes care of the modal field variation given by the modal numbers m, n, and anisotropy in the substrate. It further accounts for the dielectric, conductor, surface-wave, and radiation losses. The $\varepsilon^*_{r\,dyn}$ is computed by [6], [7],

$$\varepsilon^*_{r\,dyn} = \frac{C^*_{d\,yn}\left(\varepsilon^*_{req}, h_{eq}\right)}{C^*_{dyn}\left(\varepsilon^*_{req} = 1, h_{eq}\right)}. \tag{3}$$

All above mentioned losses in the patch are accommodated by the concept of effective loss-tangent ($\tan\delta_{eff}$). Thus, the complex equivalent relative permittivity of lossy substrate is $\varepsilon^*_{r\,eq} =$

$\varepsilon'_{r\,eq} - j\varepsilon''_{req}$, where $\varepsilon''_{req} = \varepsilon'_{req} \tan\delta_{eff}$. The $\varepsilon'_{r\,eq}$ for an anisotropic substrate is determined from (1).

The accurate computation of resonance frequency is done in three steps.

Step I: At first, we compute the approximate resonance frequency of a patch on the loss-less ($\tan\delta_{eff} = 0$) anisotropic substrate by (1) and (2).

Step II: Next, we compute all Q-factors of the patch in order to determine $\tan\delta_{eff}$. The Q_d, Q_c, Q_{sw}, and Q_r i.e., the Q-factors, due to dielectric loss, conductor loss (in patch, ground plane and in the co-axial feed), surface-wave loss, and radiation loss, respectively, are determined by the following expressions:

$$Q_d = \frac{\pi\sqrt{\varepsilon_{eff\,av}}}{\lambda_0 \alpha_d}, \qquad Q_c = \frac{\pi\sqrt{\varepsilon_{eff\,av}}}{\lambda_0\left(\alpha_c + \alpha_{feed}\right)},$$

$$Q_r = \frac{\pi}{4G_r Z\left(\varepsilon_r, w/h\right)}, \qquad Q_{sw} = \frac{Q_r \eta}{(1-\eta)} \tag{4}$$

where

$$\varepsilon_{eff\,av} = \frac{\varepsilon_{eff(w)} + \varepsilon_{eff(L)}}{2}$$

and

$$\eta = \frac{P_{sp}}{P_{sp} + P_{sw}}. \tag{5}$$

Finally, we get $\tan\delta_{eff}$ of the patch antenna

$$\tan\delta_{eff} = \frac{1}{Q_t} = \left[\frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_{sw}} + \frac{1}{Q_r}\right]. \tag{6}$$

Once total Q-factor is known, we can also compute the 1 : 2 VSWR bandwidth of the patch antenna,

$$\%BW = \frac{100}{\sqrt{2}Q_t}. \tag{7}$$

The effective permittivities $\varepsilon_{eff(W)}$ and $\varepsilon_{eff(L)}$ have been determined from the width (w) and length (L) sides respectively by the variational method [6], [7]. The dielectric loss (α_d) is determined by using the standard expression and the conductor loss (α_c) in patch and ground plane is determined from the Wheeler's inductance rule [9]. The loss in the probe feed (α_{feed}) is determined by following Collin [10]. This loss is small and can be neglected. The radiation conductance (G_r) of the radiating aperture is computed from the approximate expressions of James *et al.* [11]. The power in surface wave (P_{sw}) and power in the radiated space-wave (P_{sp}) are computed from the expressions of Pozar [12]. However, these expressions are valid for the infinite substrate which do not account for the standing wave inside the substrate.

Step III: The third step is to get the imaginary part of equivalent relative permittivity, i.e., ε''_{req} and then to compute the complex $\varepsilon^*_{r\,dyn}$. Equation (1) finally computes accurate resonance frequency the patch. The bandwidth is also computed at the final resonance frequency.

III. Experimental Setup

Chang *et al.* and Kara have used different methods for measuring the resonance frequency of the rectangular patch on the thick PTFE substrates. Chang *et al.* have kept distance of the probe (1.5 mm) fixed from edge of the patch. Thus, in their case, at the resonance frequency, the imaginary part of input impedance is not zero. They have obtained the resonance frequency of the patch corresponding to the maximum value of real part of the input impedance. Kara has measured the resonance frequency after experimentally optimizing the probe distance to get the return loss better than 30 dB. Both sets of experimental investigations do not mention about the anisotropy, loss tangent, and conductor thickness. We have taken this information from the handbook in [13]. Both the investigators have provided experimental results on the bandwidth also.

TABLE I
PATCHES OF CHANG *et al.* [1]: [NONWOVEN GLASS MICROBFIBER REINFORCED SUBSTRATE, ANISOTROPY RATIO $= 1.05$, $\varepsilon_r = 2.33 \pm 0.02$, $\tan\delta = 0.001, t = 0.005$ mm]

Patch No.	W (mm)	L (mm)	h (mm)	h/λ_g	Expt. [1] f_r (GHz)	PCAAD [4] f_r (GHz)	James [1] f_r (GHz)	Hamm. [1] f_r (GHz)	MCM [12] f_r (GHz)	Ensem. [5] f_r (GHz)	MWM f_r (GHz)
1	57.0	38.0	3.175	0.037	2.31	2.37	2.30	2.38	2.38	2.38	2.34
2	45.5	30.5	3.175	0.047	2.89	2.90	2.79	2.90	2.90	2.90	2.88
3	17.0	11.0	1.524	0.061	7.87	7.74	7.46	7.84	7.80	7.80	7.88
4	29.5	19.5	3.175	0.068	4.24	4.27	4.11	4.34	4.30	4.37	4.25
5	19.5	13.0	3.175	0.094	5.84	5.94	5.70	6.12	6.02	6.11	5.90
6	17.0	11.0	3.175	0.110	6.80	6.74	6.47	7.01	6.87	6.94	6.81
7	14.0	9.0	3.175	0.125	7.70	7.81	7.46	8.19	8.01	8.79	7.72
8	12.0	8.0	3.175	0.141	8.27	8.51	8.13	9.01	8.78	9.74	8.27
9	10.5	7.0	3.175	0.148	9.14	9.32	8.89	9.97	9.66	10.70	9.17
10	9.0	6.0	3.175	0.166	10.25	10.31	9.82	11.18	10.76	11.66	10.31
11	17.0	11.0	9.525	0.229	4.73	4.58	4.32	5.27	4.78	4.3	4.65
Maximum % Deviation :						3.17	8.67	11.42	6.17	17.78	1.69
Average of absolute % Deviation :						1.63	3.62	5.35	2.96	7.81	0.54
RMS % Deviation :						0.56	1.26	1.88	1.07	3.05	0.23

TABLE II
PATCHES OF KARA [2]: [WOVEN TYPE PTFE SUBSTRATE, ANISOTROPY RATIO $= 1.2$, $\varepsilon_r = 2.55 \pm 0.05, \tan\delta = 0.002, t = 0.005$ mm]

Patch No.	W (mm)	L (mm)	h (mm)	h/λ_g	Expt.[2] f_r (GHz)	PCAAD [4] f_r (GHz)	MCM [14] f_r (GHz)	Ensem. [5] f_r (GHz).	MWM f_r (GHz)
1	10.80	7.76	3.30	0.1405	8.00	8.409	9.010	9.697	7.835
2	14.50	9.87	4.50	0.1454	6.07	6.439	6.863	8.258	5.964
3	15.20	10.00	4.76	0.1475	5.82	6.249	6.638	7.940	5.778
4	12.55	7.90	4.00	0.1519	7.134	7.718	8.165	9.293	7.092
5	19.70	12.00	6.26	0.1553	4.66	5.015	5.215	5.030	4.606
6	14.40	8.14	4.76	0.1617	6.38	7.046	7.330	7.833	6.433
7	16.20	7.90	5.50	0.1754	5.99	6.685	6.980	5.950	6.060
8	27.56	12.56	9.52	0.1814	3.58	4.030	3.920	3.748	3.626
9	26.40	10.20	9.52	0.1976	3.90	4.439	3.938	3.895	3.956
10	26.20	9.74	9.52	0.2017	3.98	4.528	3.965	3.895	4.018
11	35.00	12.65	12.81	0.2032	2.98	3.416	2.958	2.920	3.031
12	23.00	7.83	8.45	0.2091	4.60	5.312	4.348	4.545	4.691
13	26.76	8.83	10.00	0.2119	3.98	4.580	3.800	3.814	4.027
14	34.00	10.80	12.81	0.2148	3.15	3.640	2.973	2.972	3.193
15	33.80	10.30	12.81	0.2182	3.20	3.705	2.980	3.005	3.242
16	31.30	9.20	12.00	0.2216	3.47	4.026	3.225	3.211	3.507
17	28.35	7.77	11.0	0.2284	3.90	4.519	3.388	3.563	3.909
Maximum % Deviation :						16.02	16.53	36.43	2.06
Average of absolute % Deviation :						12.06	8.93	11.62	1.22
rms % Deviation :						3.06	2.51	4.08	0.32

IV. Results and Conclusion

Tables I and II show the experimental resonance frequency of the rectangular patches of Chang *et al.* [1] and Kara [2], respectively. The tables also contain the computed resonance fre-

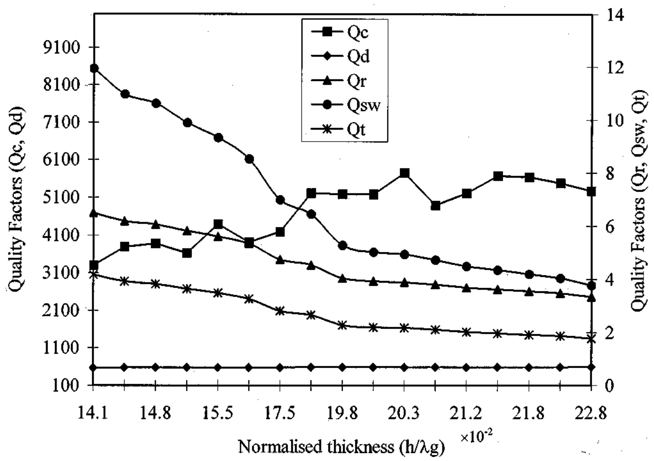

Fig. 2. Q-Factors due to conductor, dielectric, radiation, surface wave and total losses of patches of Kara shown in Table II.

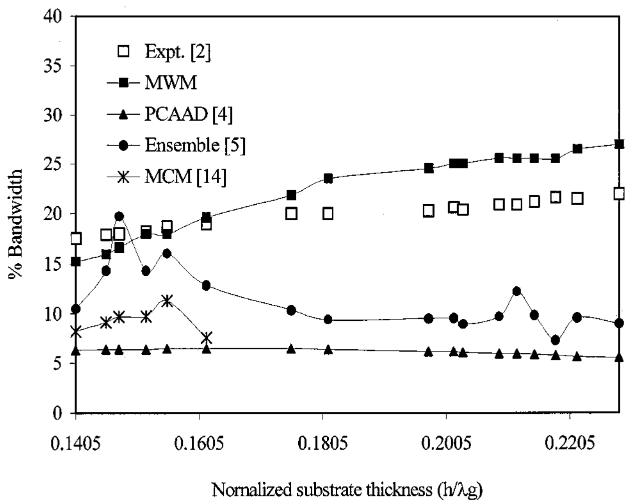

Fig. 3. % bandwidth with electrical thickness of patches in Table II.

quency by the cavity model of Richards *et al.*, as adopted by Pozar in his commercial software PCAAD [4], transmission line model of James [1], Hammerstad [1], multiport cavity model (MCM) of Benalla *et al.* [14], MOM of Ensemble [5], and, finally, the present version of the modified Wolff model (MWM).

It is obvious that the present form of the MWM is much better than the previous form of the cavity model, the transmission models, the multiport cavity model (MCM), and, finally, the MOM based commercial software, Ensemble. The MCM and Ensemble have deviation as high as 16% and 36%, respectively. The MWM has maximum deviation within 2% and average deviation 0.54%. Thus, the anisotropy and losses, mainly surface-wave and radiation losses, are key factors in influencing the resonance frequency of a patch radiator on the thick substrate which have not been accounted for in these models. It would be interesting to compare these experimental results against the finite element method (FEM) and finite difference time domain (FDTD) based commercial software. However, we do not posses these software to do the same. In Figs. 2 and 3, we present the Q-factors and the bandwidth for the patches of Kara. Fig. 2 clearly shows the role of each Q-factor for a patch radiator on the thick substrate. The low Q-factor due to the surface wave loss decreases the efficiency to 60%, and it also degrades the radiation pattern. Finally, Fig. 3 compares the bandwidth computed by the above-mentioned models against the experimental bandwidth of Kara. The MWM shows better agreement with the experimental results as compared to models and software mention in this paper. For thickness above $0.1605\lambda_g$, the MCM does not work properly.

ACKNOWLEDGMENT

A. K. Verma expresses his sincere thanks to the Nanyang Technological University, Singapore, for the award of the Tan Chin Tuan Exchange Fellowship. This work was completed at NTU, Singapore and A. K. Verma is thankful to Dr. C. L. Law for taking keen interest in the work.

REFERENCES

[1] E. Chang, S. A. Long, and W. F. Richards, “An experimental investigation of electrically thick rectangular microstrip antennas,” *IEEE Trans, Antennas Propagat.*, vol. AP–34, pp. 767–772, June 1986.
[2] M. Kara, “A novel technique to calculate the bandwidth of rectangular microstrip antenna elements with thick substrate,” *Microwave Opt. Technol. Lett.*, vol. 12, pp. 59–64, June 1996.
[3] R. Garg and S. A. Long, “Resonant frequency of electrically thick rectangular microstrip antennas,” *Electron. Lett.*, vol. 23, pp. 1149–1151, Oct. 1987.
[4] D. M. Pozar, *PCAAD 3.0: Personal Computer Aided Antenna Design*. Leverett, MA: Antenna Design Associates, Inc., 1996.
[5] *Ensemble Version 6.1*. Boulder, CO: Ansoft Co., Sept. 1999.
[6] A. K. Verma and Z. Rostamy, “Resonance frequency of uncovered and covered rectangular microstrip patch using modified Wolff model,” *IEEE Trans., Microwave Theory Tech.*, vol. 41, pp. 109–116, Jan. 1993.
[7] A. K. Verma and A. Bhupal, “Resonance frequency of anisotropic multilayer rectangular microstrip patch using MWM,” *Electron. Lett.*, vol. 34, pp. 323–325, Feb. 1998.
[8] B. Bhat and S. K. Koul, “New approach to analyze strip- and microstrip like transmission lines with anisotropic substrates having a tilted optical axis,” in *Proc. Inst. Elect. Eng., MOA*, vol. 131, June 1984, pp. 191–197.
[9] A. K. Verma and A. Bhupal, “Conductor loss of multilayer microstrip line using single layer reduction formulation,” *Microwave Opt. Technol. Lett.*, vol. 19, pp. 20–24, Sept. 1998.
[10] R. E. Collin, *Antennas and Radiowave Propagation*. New York: McGraw-Hill, 1985, pp. 273–283.
[11] J. R. James, A. Henderson, and P. Hall, “Microstrip antenna performance is determined by substrate constraints,” *MSN*, pp. 73–83, Aug. 1982.
[12] D. M. Pozar, “Rigorous closed-form expressions for the surface wave loss of printed antennas,” *Electron. Lett.*, vol. 26, pp. 954–956, June 1990.
[13] R. Garg, P. Bhartia, I. Bahl, and A. Ittipiboon, *Microstrip Antenna Design Handbook*. Norwood, MA: Artech House, 2000, pp. 759–770.
[14] A. Benalla, C. H. Thng, and K. C. Gupta, “Computer-aided design and analysis of microstrip patch antennas,” *Micropatch Version 2.0*, 1993.